Mathematics
Course of Study – Algebra II
2024

THE DIOCESE OF COLUMBUS

Adam J. Dufault, Ed.D., Superintendent of Schools

Daphne Irby, Associate Director for Teaching and Learning

Office of Catholic Schools
Diocese of Columbus

Members of the Mathematics Course of Study Committee

The Office of Catholic schools would like to thank the members of the Mathematics Course of Study Committee.

Elementary Schools

Abby Petrozzi	St. Paul School
Audriana Lindamood	Notre Dame School
Eriny Attia	St Michael School
Joyce Kemmerly	St. Paul School
Kerinan Jordan	St. Brigid of Kildare School
Julie Hedrick	St. Cecilia School
Julie Zucker	St. Paul School
Katharine Gaddis	St. Brendan the Navigator School
Kristen Griffin-Pierce	St. Joseph Montessori School
Kristin Jarvis	Our Lady of Perpetual Help School
Linda Cotter	St. Mary School – German Village
Lynn Schwalm	St. James the Less School
Michelle Baltikauskas	Our Lady of Perpetual Help School
Molly Zesch	St. Agatha School
Paula Thompson	St. Brigid of Kildare School
Rebekah Springer	Holy Spirit School
Stephanie Speed	St. Paul School
Sue McFadden	St. Matthew the Apostle School
Teri Baum	Our Lady of Perpetual Help School
Joy Bair	St. Brendan School

High Schools

Jean Garrick	St. Francis DeSales High School
Julie Fix	Bishop Hartley High School
Melissa Martin	Notre Dame Jr./Sr. High School
Sheila Yerkey	St. Francis DeSales High School

Table of Contents

INTRODUCTION

The following is the revised Mathematics Course of Study for the Catholic Diocese of Columbus. The committee has used the new Ohio Learning Standards for mathematics adopted by the State of Ohio in 2017 as the foundation of this Course of Study.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO OUR FAITH

The goal of mathematics education in a Catholic school is to produce mathematically literate, faithful individuals who can not only function in a global world of increasing complexity but who can also, more importantly, understand and speak the truth of God's teachings. To meet this challenging goal, the students in the Diocese of Columbus Catholic Schools will need to develop math content knowledge, apply their knowledge and skills, reason logically, think critically, solve problems creatively, resourcefully and morally, and communicate with others effectively, all within the context of our faith.

As Dr. Brett Salkeld states in his book, *Educating for Eternity*, "… an authentic Catholic education forms people who can change the world, not in spite of their desire for God and their hope for heaven, but because of it."[1] The need to understand and to be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. Those who accomplish this goal will significantly enhance their opportunities for shaping their future and the future of others. In keeping with this goal, students are presented with a sequential development of mathematical concepts. "Math is a unique and privileged way of perceiving the truth, goodness, and beauty of God's creation. It is therefore, a path to God himself."[1]

To best support the students, the classroom environment should foster enthusiastic learning and appreciation for the power, beauty, and usefulness of mathematics. Students will see mathematics as "both deeply mysterious and deeply intelligible,"[1] like God himself. They will see math reveal truths about creation and in doing so, reveals truth about the Creator. Math is an interdisciplinary problem-solving tool, a universal language, an art, and a powerful mechanism to further God's plan for humankind.

Effective teaching in a Catholic school engages students in application and cultivation of virtue into the math practices. Below are listed some examples of how a student can practice the virtues in mathematics.

Student will practice <u>perseverance</u>

- Continuing to work, willingly, even when the work is hard
- "Let's work through it step by step"
- "You can do hard things" (Philippians 4:13 - I can do all things through Christ which strengthens me)
- "We do not have a 'magic wand" for everything, but we do have trust in the Lord who accompanies us and never abandons us" (Pope Francis)

Students will practice <u>docility</u>

- Error analysis
- Graciously accepting feedback
- Proverbs 9:9 -"Instruct a wise man and he will be wiser still; teach a

[1] Salkeld, Brett. *Eucating for Eternity, A Teacher's Companion for Making Every Class Catholic*. Our Sunday Visitor, 2023.

Back to Index

righteous man and he will add to his learning."

Student will practice humility

- Being able to recognize that you need help and asking for help
- Not bragging that things are "too easy" and recognizing that others may need more time.
- Ephesians 4:2 – "Be completely humble and gentle; be patient, bearing with one another in love"

Student will practice meekness

- Staying in control of your actions and attitude
- Knowing how your attitude and actions impact others
- James 4:6- "But He gives a greater grace. Therefore, it says, "God is opposed to the proud, but gives grace to the humble."

PRINCIPLES FOR MATHEMATICS FOR THE DIOCESE OF COLUMBUS

Equity. Excellence in mathematics education requires equity – high expectations based on the standards which should be accessible to all students, regardless of learning differences.

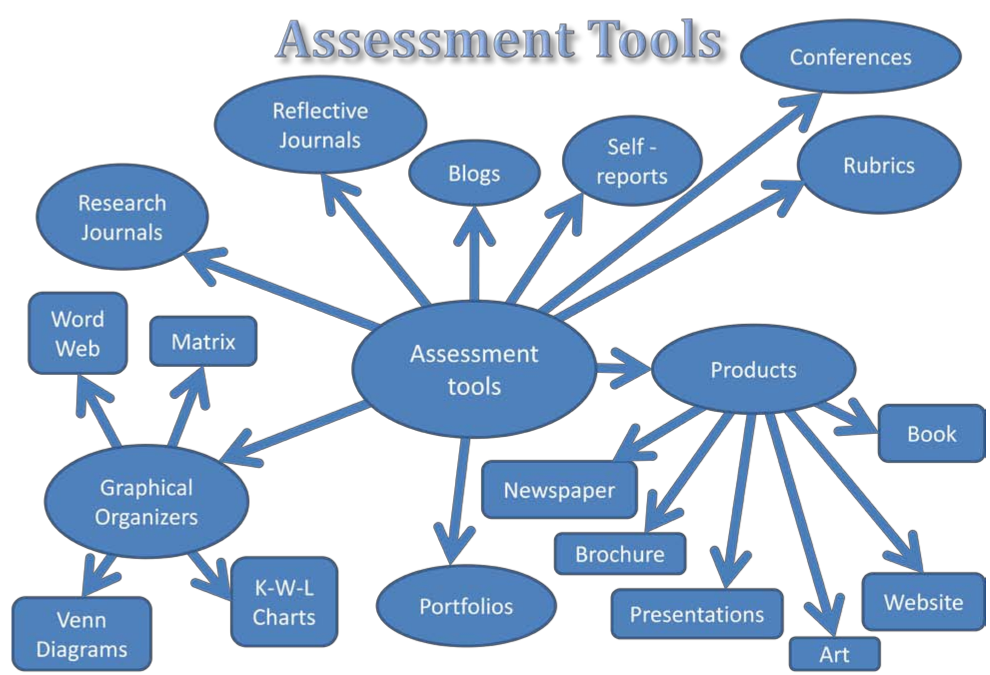
Curriculum. A curriculum is more than a collection of activities. It must be coherent, focused, well-articulated, and integrated with our Catholic values.

Teaching. Effective mathematics teaching requires a student-teacher relationship. Educators understanding what students know and need to learn and be able to do, and are supportive in moving students along the continuum of learning.

Learning. Students not only learn but understand mathematics with by actively building new knowledge from prior knowledge and experiences.

Technology. Technology is useful in teaching and learning mathematics. As any tool, technology can enhance students' learning while not substituting for the teacher and the student-teacher relationship.

Assessment. Multiple and appropriate assessments should align to the Course of Study and support the learning of important mathematics, be formative as well as summative, and furnish useful information to teachers, students and parents. Assessment results should guide teachers' instruction and interventions as well as grade promotion decisions. Useful assessments align to the standards in the Course of Study both in what a student needs to know and be able to do, and should match what the student is expected to learn. There are many tools (e.g. portfolios, rubrics, interviews) other than the standard paper and pencil tests to assess a student's understanding of



the material.

Back to Index

One method that has continued to increase student achievement is involving them in all steps of the assessment process. At the most basic level, students should understand how their grades will be determined. As assessment becomes more student-centered, the students can develop rubrics, maintain their own assessment records, self-assess, and communicate their achievement to others (student-led conferences).

PROCESS

The Ohio Learning Standards, which are the basis of the Diocese of Columbus' Course of Study, were developed through a feedback and revision process.

After the learning standards were adopted by the State Board of Education, a Diocesan committee was formed to review and adjust these standards so that the Course of Study reflects and integrates the Catholic faith and traditions.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

DIFFERENTIATION

The content standards are grade and course-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should understand the standards and sequential order of mathematics as to allow for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of all students. For example, schools should allow students with a visual disability to use Braille, audio technology or other assistive devices for reading and a scribe, or speech-to-text technology for writing. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 8 with the Diocesan eight Standards for Mathematical Practices for Students which were adapted from the Ohio Mathematical Practices to include the Catholic virtues. Also included are a set of Teaching Practices from the National Council of Teachers of Mathematics (NCTM) on page 7.

Effective Mathematics Teaching Practices[2]
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear learning goals, situates goals within learning progressions, and used the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematicsengages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allows for multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations, deepening understanding of mathematics concepts and procedures. A Catholic math class should be a place where students learn to rejoice in God's truth.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing approaches and arguments. As they work together, students should exhibit the virtue of magnanimity or benevolence, keeping the greater good in the forefront.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, becomes skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistentlyprovides students, individually and collectively, with opportunities and supports to engage in productivestruggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of studentthinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning (e.g. "Show me your work!")

[2] NCTM (National Council of Teachers of Mathematics)

Mathematical Practices for Students with Connections to Catholic Virtues

Make sense of problems and persevere in solving them. Mathematically proficient students strive to understand the meaning of a problem and look for ways to solve it. They monitor, and evaluate their progress and change course if necessary. They conduct error analysis and learn from their mistakes. They demonstrate the virtues of perseverance as they work through the problem. They check their answers to problems using different methods and they continually ask themselves, “Does this make sense?”

Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They understand that symbols are used to represent quantities. They can represent a problem symbolically and manipulate the representing symbols to solve the problem. They can make connections between mathematical representations and the physical world. They understand that those representations reveal truths about creation and about God.

Construct viable arguments and critique the reasoning of others. Mathematically proficient students can listen to the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. They can justify their conclusions, communicate them to others, and respond to the arguments of others. They can use logic and reasoning to compare the effectiveness of two plausible arguments. If there is a flaw in an argument, they can use reasoning and logic to explain what it is. They demonstrate the virtues of temperance and humility in making their arguments and critiquing others.

Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can use mathematics to advance Catholic social teachings.

Mathematical Practices for Students (cont'd)
Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts when appropriate.
Attend to precision. Mathematically proficient students communicate precisely to others. They use clear definitions and mathematical language in discussion with others and in their own reasoning. They demonstrate self-control and humility in their communications.
Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. They also step back for an overview and shift perspective when needed. They understand that math is orderly and intelligible and a way to reveal truths about God and creation. They demonstrate the virtue of circumspection as they ponder the truths of creation.
Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for appropriate shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. They continue to demonstrate the virtues of prudence, fortitude and temperance while solving problems.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. A lack of understanding of the content effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice.

Noteworthy Changes From 2018 To 2024

There have been some changes to the Mathematics Course of Study in 2024. These changes are detailed out below:

Geometry

1. Congruence and proof of triangles were separated from construction and transformations as a separate critical area of focus.
2. Understanding and using line relationships was added.
3. Completing the square to find the center and radius of a circle was added as an extension to GPE.1.
4. Law of Sines and Cosines were added from Algebra II.

Algebra I

1. The Real Number standards (RN.1, RN.2amd RN.3) have been moved back into the Algebra I (from Algebra II) course of study with this additional note: The focus of these standards should be on the properties of square roots, simplifying square roots and the arithmetic operations of square roots (add, subtract, multiply).

Algebra II

1. Using the Law of Sines and Cosines to solve problems and to find unknown measurements in right triangles were moved to Geometry; however, finding unknow measurements in non-right triangles and extending right triangle trigonometry to include obtuse angles was left in Algebra II.
2. *Extend to include solving systems of linear equations using matrices, with the option to solve with technology* was added as an extension to A.REI.11.

Mathematical Content Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> *(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).*

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students. However, standards with a (+) symbol are considered over and above what is expected for that course.

The high school standards are listed in conceptual categories:

- Modeling
- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Proofs in high school mathematics should not be limited to geometry. Mathematically proficient high school students employ multiple proof methods, including algebraic derivations, proofs using coordinates, and proofs based on geometric transformations, including symmetries. These proofs are supported by the use of diagrams and dynamic software and are written in multiple formats including not just two-column proofs but also proofs in paragraph form, including mathematical symbols. In statistics, rather than using mathematical proofs, arguments are made based on empirical evidence within a properly designed statistical investigation.

HOW TO READ THE HIGH SCHOOL CONTENT STANDARDS

Conceptual Categories are areas of mathematics that cross through various course boundaries.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Standards define what students should understand and be able to do.

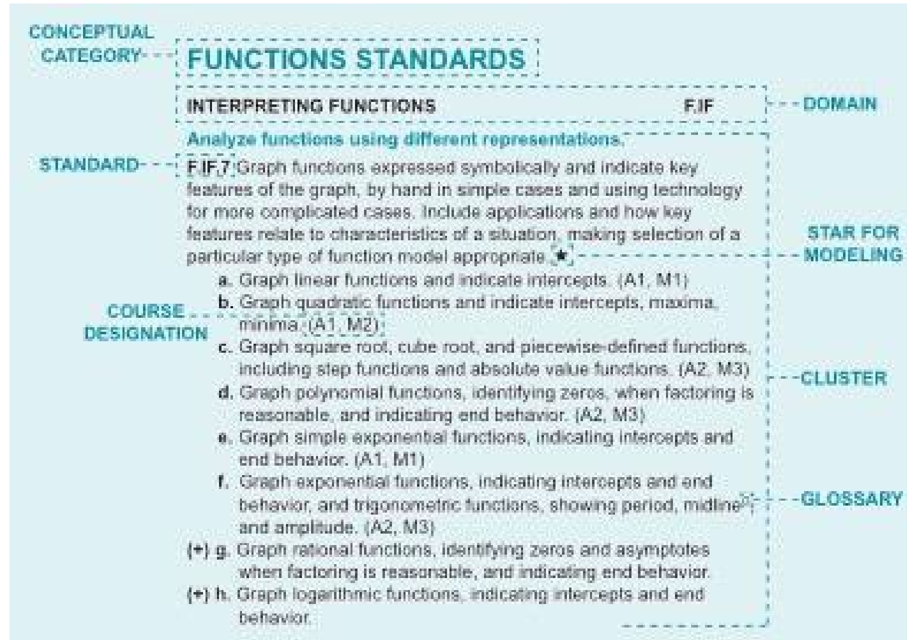
G shows there is a definition in the glossary for this term.

(★) indicates that modeling should be incorporated into the standard. (See the Conceptual Category of Modeling pages 12-13)

(+) indicates that it is a standard for students who are planning on taking advanced courses. Standards with a (+) sign will not appear on Ohio's State Tests.

Some standards have course designations such as (A1, M1) or (A2, M3) listed after an **a., b.,** or **c.** These designations help teachers know where to focus their instruction within the standard. In the example below the beginning section of the standard is the stem. The stem shows what the teacher should be doing for all courses. (Notice in the example below that modeling (★) should also be incorporated.) Looking at the course designations, an Algebra 1 teacher should be focusing his or her instruction on **a.** which focuses on linear functions; **b.** which focuses on quadratic functions; and **e.** which focuses on simple exponential functions. An Algebra 1 teacher can ignore **c., d.,** and **f,** as the focuses of these types of functions will come in later courses. However, a teacher may choose to touch on these types of functions to extend a topic if he or she wishes.

Notice that in the standard below, the stem has a course designation. This shows that the full extent of the stem is intended for an Algebra 2 or Math 3 course. However, **a.** shows that Algebra 1 and Math 2 students are responsible for a modified version of the stem that focuses on transformations of quadratics functions and excludes the *f(kx)* transformation. However, again a teacher may choose to touch on different types of functions besides quadratics to extend a topic if he or she wishes if he or she wishes.



Back to Index

High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

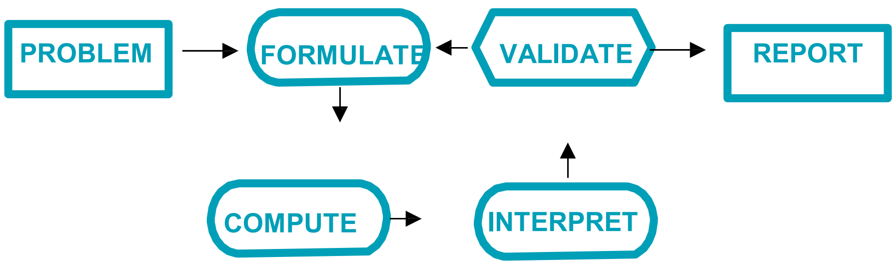
In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from



a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

MODELING STANDARDS

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★)

Algebra II Critical Areas of Focus

CRITICAL AREA OF FOCUS #1

Inferences and Conclusions from Data

Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn. To model the relationships between variables, students fit a linear function to data and analyze residuals to assess the appropriateness of fit. Building on prior experiences associated with the notion of a correlation coefficient, students learn how to distinguish a statistical relationship from a cause-and-effect relationship.

CRITICAL AREA OF FOCUS #2

Polynomials, Rational and Radical Relationships

Students apply their new understanding of number to seeing structure in exponential expressions. Students develop the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. This learning culminates with applying the Fundamental Theorem of Algebra. A central theme of the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students realize that rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, they learn that rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. In addition, students will extend their experience with solving systems of two linear equations in two variables to solving systems of three linear equations in three variables algebraically.

CRITICAL AREA OF FOCUS #3

Trigonometry of General Triangles and Trigonometric Functions

Building on students' previous work with functions, trigonometric ratios, and circles in Geometry or Mathematics 2, students now use the coordinate plane and the unit circle to extend trigonometry to general angles and to model periodic phenomena. This leads to the conclusion that trigonometry is applied beyond the right triangle—that is, at least to obtuse angles. Concurrently, students develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They also develop an understanding that the Laws of Sines and Cosines can be used to find missing measures of general (not necessarily right) triangles. This allows students to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

CRITICAL AREA OF FOCUS #4

Modeling with Functions

Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Algebra 2 Course Overview

NUMBER AND QUANTITY

THE COMPLEX NUMBER SYSTEM

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.

ALGEBRA

SEEING STRUCTURE IN EXPRESSIONS

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

CREATING EQUATIONS

- Create equations that describe numbers or relationships.

REASONING WITH EQUATIONS AND INEQUALITIES

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.
- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

FUNCTIONS

BUILDING FUNCTIONS

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

- Construct and compare linear, quadratic, and exponential models, and solve problems.

TRIGONOMETRIC FUNCTIONS

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

GEOMETRY

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

- Define trigonometric ratios, and solve problems involving right triangles.
- Apply trigonometry to general triangles.

CIRCLES

- Find arc lengths and areas of sectors of circles.

Algebra 2, Course Overview, continued

STATISTICS AND PROBABILITY

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models.

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments and observational studies.

High School—Number and Quantity

NUMBERS AND NUMBER SYSTEMS

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole number exponents leads to new and productive notation. For example, properties of whole number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

QUANTITIES

In real-world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Standards

THE COMPLEX NUMBER SYSTEM

N.CN.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a + bi* with *a* and *b* real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

(+) **N.CN.8** Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. (+) **N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Back to Index

High School—Algebra

EXPRESSIONS

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

EQUATIONS AND INEQUALITIES

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

Back to Index

High School—Algebra, CONTINUED

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = (\frac{(b1+b2)}{2})h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

CONNECTIONS WITH FUNCTIONS AND MODELING

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Back to Index

Algebra Standards

SEEING STRUCTURE IN EXPRESSIONS

Interpret the structure of expressions.

A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, to factor 3x(x − 5) + 2(x − 5), students should recognize that the "x − 5" is common to both expressions being added, so it simplifies to (3x + 2)(x − 5); or see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*

Write expressions in equivalent forms to solve problems.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

c. Use the properties of exponents to transform expressions for exponential functions. *For example,* 8^t *can be written as* 2^{3t}*.*

(+) **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS

Perform arithmetic operations on polynomials.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

b. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)

Understand the relationship between zeros and factors of polynomials.

A.APR.2 Understand and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by *x − a* is *p(a).* In particular, *p(a) = 0* if and only if *(x – a)* is a factor of *p(x).*

A.APR.3 Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

(+) **A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers. *For example by using coefficients determined for by Pascal's Triangle.* the Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Back to Index

Algebra Standards, CONTINUED

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS

Rewrite rational expressions.

A.APR.6 Rewrite simple rational expressions[G] in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

CREATING EQUATIONS

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★

c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

c. Extend to include more complicated function situations with the option to graph with technology. (A2, M3)

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1)

a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★

d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

REASONING WITH EQUATIONS AND INEQUALITIES

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve systems of equations.

A.REI.6 Solve systems of linear equations algebraically and graphically.

b. Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)

(+) A.REI.7 Extend to include solving systems of linear equations using matrices, with the option to solve with technology.

Represent and solve equations and inequalities graphically.

A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = {}^{100}/_{v}$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph, e.g., the trace of a seismograph; by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

CONNECTIONS TO EXPRESSIONS, EQUATIONS, MODELING, AND COORDINATES.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Standards

INTERPRETING FUNCTIONS

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★(A2, M3)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

c. Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★

c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3)

d. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3)

f. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline[G], and amplitude. (A2, M3)

(+) **g.** Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior. (A2, M3) (+) **h.** Graph logarithmic functions, indicating intercepts and end behavior. (A2, M3)

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3)

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change[G] in functions such as* $y = (1.02)^t$*, and* $y = (0.97)^t$ *and classify them as representing exponential growth or decay.* (A2, M3)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)

Functions Standards, CONTINUED

BUILDING FUNCTIONS

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations.

For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3)

c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time

Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)

F.BF.4 Find inverse functions.

b. Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3)

c. Verify by composition that one function is the inverse of another. (A2, M3)

d. Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★

High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non- right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

CONNECTIONS TO EQUATIONS

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Back to Index

Geometry Standards

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

Define trigonometric ratios, and solve problems involving right triangles.

G.SRT.8 Solve problems involving right triangles.★

(+) **b.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ (A2, M3)

Apply trigonometry to general triangles.

G.SRT.9 Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G.SRT.10 Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.

CIRCLES

Find arc lengths and areas of sectors of circles.

G.C.6 Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)

High School—Statistics and Probability

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

CONNECTIONS TO FUNCTIONS AND MODELING

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Standards

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

Understand independence and conditional probability, and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★

S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having a dog and being a dog lover with the chance of being a cat lover and having a cat.* ★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.★

S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★

(+) **S.CP.8** Apply the general Multiplication Rule in a uniform probability model[G], P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.★ (G, M2)

(+) **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.★ (G, M2)

Back to Index

High School Math Appendix- Curriculum Integration

1. History of a mathematician
    a. Investigate past or current mathematician with a focus on saint examples
    b. Include historical background
    c. State their contributions.
    d. Give their mathematical & historical importance.
    e. How do we see their work used today.
    f. Students share why they picked the person, and what was the most valuable thing they learned.
2. Vocabulary Scavenger Hunt
    a. Terms are posted around the room.
        i. Students must come up with images to illustrate.
        ii. Students must also define the terms.
    b. Images are provided to students.
        i. Students find and name math found in the images.
        ii. Students must also define the terms.
3. Financial Growth/Decay situations
    a. Study the decreasing value of a car.
    b. Study credit cards & mortgages for real value price paid.
    c. Students find how much to invest at current rates in order to save for something in the near future (i.e. car, trip, etc.).
    d. Study population growth and its impact on our world today.
4. Project involving the Golden Ratio
5. Project involving math in art
6. Project involving math in nature

Back to Index

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: ¾ and $-\frac{3}{4}$ are additive inverses of one another because ¾ $+ (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$.

Adjacent Angles.

Algorithm. *See also:* computation algorithm.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1] *See also:* first quartile and third quartile.

Circumference.

Clusters. A number of similar things growing, collected or grouped together. Clusters can be used to break large collections of data into smaller groups.

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Complementary Angles.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* **computation strategy**.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dependent Variable. A variable that changes when the value of another variable changes. It is something that depends on other factors.

Diameter.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. **the**

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[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/ standards/mathglos.html, accessed March 2, 2010.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[4] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Back to Index

process of resizing an object by a increasing or decreasing the dimensions by certain scale factor can be "scaled" up or down

Discount.

Dividend. The number or quantity being divided (split into groups)

Divisor. The quantity that determines the number of equal parts or groups the dividend is split into

Dot plot. *See also:* line plot.

Expanded form. A multi- digit number is expressed in expanded form when it is written as a sum of single- digit multiples of powers of ten. For example,
643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

Expression. A math phrase that contains numbers, operations, and/or variables.

Equation. Two or more expressions that are equal to one another.

Factor.

First quartile. For a data set with median M, the first quartile is the median of
the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fluency in algorithms -. The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

Fluency in math facts- Be able to reclass math facts with automaticity.

Fluently. *See also:*
fluency.

Fraction. A number expressible in the form $^a/b$ where *a* is a whole number and *b* is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Frequency.

Gaps. An interval where there are no data points present.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Independent Variable. A variable that does not depend on any other variable for its value. It stands alone and isn't changed by the other variables you are trying to measure.

Inequality. A relationship between two expressions or values that are not equal to each other.

Integer. A number expressible in the form *a* or −*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12,
14, 15, 22, 120}, the interquartile range is
15 − 6 = 9. *See also:* first quartile, third quartile.

Inverse. One of a pair of numbers that when operated on together give the identity.

Justify: To provide a convincing argument for
the truth of a statement to a particular audience.

Linear Equation. An algebraic equation where each variable is raised to the power of 1. In one or two variables, it always represents a straight line.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Markup.

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15,
22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the

number of data values.
Example: For the data set
{2, 3, 6, 7, 10, 12, 14, 15,
22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Mode.

Multiple.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Outlier.

Perfect Square.

Pi.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Population.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

Proportional.

Prove: To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

Quotient. The answer to a division problem

Radius.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Reciprocal. One of a pair of numbers or expressions whose product is one. See also **multiplicative inverse**.

Rectilinear figure. A polygon all angles of which are right angles.

Reflection.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. See also: terminating decimal.

Rotation.

Sample Population.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scaled Drawing.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[3]

Similarity transformation. A rigid motion followed by a dilation.

Skewed Data.

Supplementary Angles.

Symmetrical Data.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Transversal.

Trapezoid. 1. A trapezoid is a quadrilateral with at least one pair of parallel sides (Inclusive definition) 2. A trapezoid is a quadrilateral wit exactly one pair of parallel sides. (exclusive definition) *Districts may choose either definition to use for instruction. Ohio's State Tests' items will be written so that either definition will be acceptable.*

Uniform probability model. A probability model which assigns equal probability to all outcomes. See also: probability model.

Unit Rate. A ratio (a comparison using division) of different units whose denominator is one. Common Unit rate Miles per Gallon or Dollars per Hour

Variable. A variable is an object, event, idea, feeling, time period, or any other type of category you are trying to measure.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Verify: To check the truth or correctness of a statement in specific cases.

Vertical Angles.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

[3] Adapted from Wisconsin Department of Public Instruction