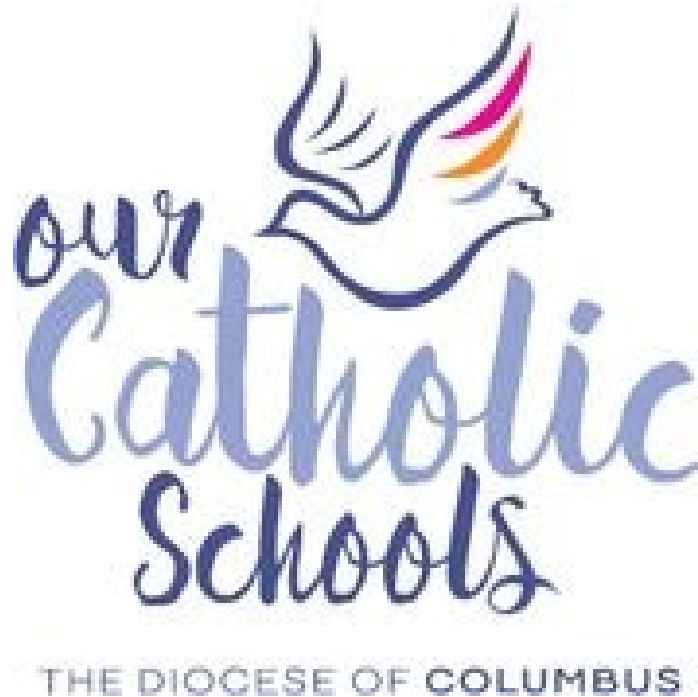


Mathematics Course of Study - Geometry 2024



Adam J. Dufault, Ed.D., Superintendent of Schools
Daphne Irby, Associate Director for Teaching and Learning

Office of Catholic Schools
Diocese of Columbus

Members of the Mathematics Course of Study Committee

The Office of Catholic schools would like to thank the members of the Mathematics Course of Study Committee.

Elementary Schools

Abby Petrozzi
Audriana Lindamood
Eriny Attia
Joyce Kemmerly
Kerinan Jordan
Julie Hedrick
Julie Zucker
Katharine Gaddis
Kristen Griffin-Pierce
Kristin Jarvis
Linda Cotter
Lynn Schwalm
Michelle Baltikauskas
Molly Zesch
Paula Thompson
Rebekah Springer
Stephanie Speed
Sue McFadden
Teri Baum
Joy Bair

St. Paul School
Notre Dame School
St Michael School
St. Paul School
St. Brigid of Kildare School
St. Cecilia School
St. Paul School
St. Brendan the Navigator School
St. Joseph Montessori School
Our Lady of Perpetual Help School
St. Mary School – German Village
St. James the Less School
Our Lady of Perpetual Help School
St. Agatha School
St. Brigid of Kildare School
Holy Spirit School
St. Paul School
St. Matthew the Apostle School
Our Lady of Perpetual Help School
St. Brendan School

High Schools

Jean Garrick
Julie Fix
Melissa Martin
Sheila Yerkey

St. Francis DeSales High School
Bishop Hartley High School
Notre Dame Jr./Sr. High School
St. Francis DeSales High School

TABLE OF CONTENTS

INTRODUCTION	4
Effective Mathematics Teaching Practices ²	7
Mathematical Practices for Students with Connections to Catholic Virtues	8
Noteworthy Changes From 2018 To 2024	11
Mathematical Content Standards for High School	12
How To Read The High School Content Standards	13
High School—Modeling.....	14
Geometry Critical Areas of Focus	16
Geometry Course Overview	18
High School—Geometry	19
Geometry Standards	21
High School—Statistics and Probability.....	25
Statistics and Probability Standards.....	26
High School Math Appendix- Curriculum Integration	27
Glossary.....	28

INTRODUCTION

The following is the revised Mathematics Course of Study for the Catholic Diocese of Columbus. The committee has used the new Ohio Learning Standards for mathematics adopted by the State of Ohio in 2017 as the foundation of this Course of Study.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO OUR FAITH

The goal of mathematics education in a Catholic school is to produce mathematically literate, faithful individuals who can not only function in a global world of increasing complexity but who can also, more importantly, understand and speak the truth of God's teachings. To meet this challenging goal, the students in the Diocese of Columbus Catholic Schools will need to develop math content knowledge, apply their knowledge and skills, reason logically, think critically, solve problems creatively, resourcefully and morally, and communicate with others effectively, all within the context of our faith.

As Dr. Brett Salkeld states in his book, *Educating for Eternity*, "... an authentic Catholic education forms people who can change the world, not in spite of their desire for God and their hope for heaven, but because of it."¹ The need to understand and to be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. Those who accomplish this goal will significantly enhance their opportunities for shaping their future and the future of others. In keeping with this goal, students are presented with a sequential development of mathematical concepts. "Math is a unique and privileged way of perceiving the truth, goodness, and beauty of God's creation. It is therefore, a path to God himself."¹

¹ Salkeld, Brett. *Educating for Eternity, A Teacher's Companion for Making Every Class Catholic*. Our Sunday Visitor, 2023.

To best support the students, the classroom environment should foster enthusiastic learning and appreciation for the power, beauty, and usefulness of mathematics. Students will see mathematics as "both deeply mysterious and deeply intelligible,"¹ like God himself. They will see math reveal truths about creation and in doing so, reveals truth about the Creator. Math is an interdisciplinary problem-solving tool, a universal language, an art, and a powerful mechanism to further God's plan for humankind.

Effective teaching in a Catholic school engages students in application and cultivation of virtue into the math practices. Below are listed some examples of how a student can practice the virtues in mathematics.

Student will practice perseverance

- Continuing to work, willingly, even when the work is hard
- "Let's work through it step by step"
- "You can do hard things" (Philippians 4:13 - I can do all things through Christ which strengthens me)
- "We do not have a 'magic wand' for everything, but we do have trust in the Lord who accompanies us and never abandons us" (Pope Francis)

Students will practice docility

- Error analysis
- Graciously accepting feedback
- Proverbs 9:9 - "Instruct a wise man and he will be wiser still; teach a righteous man and he will add to his learning."

Student will practice humility

- Being able to recognize that you need help and asking for help
- Not bragging that things are “too easy” and recognizing that others may need more time.
- Ephesians 4:2 – “Be completely humble and gentle; be patient, bearing with one another in love”

Student will practice meekness

- Staying in control of your actions and attitude
- Knowing how your attitude and actions impact others
- James 4:6- “But He gives a greater grace. Therefore, it says, “God is opposed to the proud, but gives grace to the humble.”

PRINCIPLES FOR MATHEMATICS FOR THE DIOCESE OF COLUMBUS

Equity. Excellence in mathematics education requires equity – high expectations based on the standards which should be accessible to all students, regardless of learning differences.

Curriculum. A curriculum is more than a collection of activities. It must be coherent, focused, well-articulated, and integrated with our Catholic values.

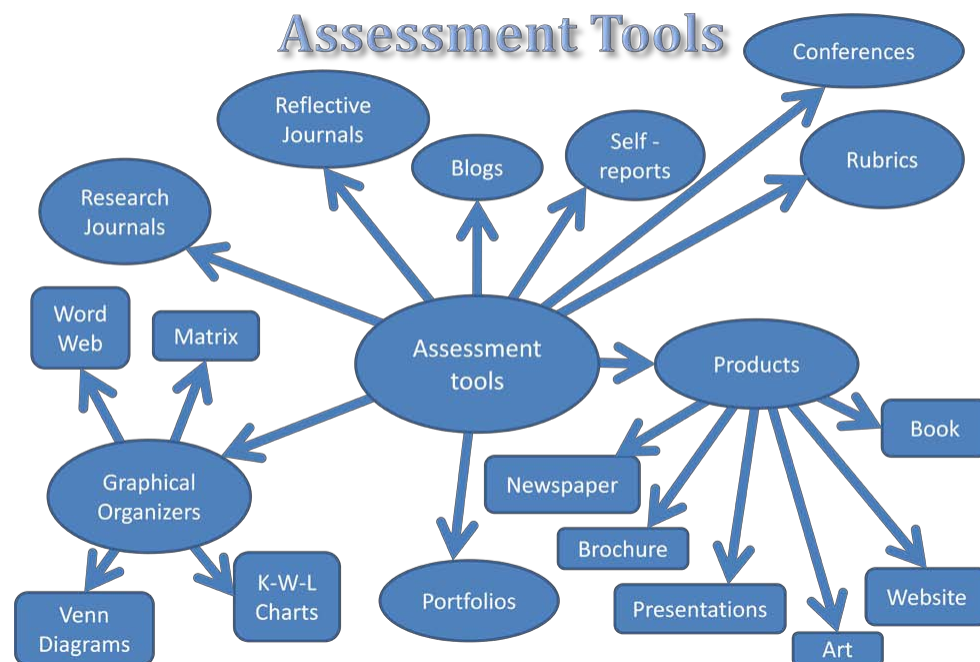
Teaching. Effective mathematics teaching requires a student-teacher relationship. Educators understanding what students know and need to learn and be able to do, and are supportive in moving students along the continuum of learning.

Learning. Students not only learn but understand mathematics with by actively building new knowledge from prior knowledge and experiences.

Technology. Technology is useful in teaching and learning mathematics. As any tool, technology can enhance students’ learning while not substituting

for the teacher and the student-teacher relationship.

Assessment. Multiple and appropriate assessments should align to the Course of Study and support the learning of important mathematics, be formative as well as summative, and furnish useful information to teachers, students and parents. Assessment results should guide teachers’ instruction and interventions as well as grade promotion decisions. Useful assessments align to the standards in the Course of Study both in what a student needs to know and be able to do, and should match what the student is expected to learn. There are many tools (e.g. portfolios, rubrics, interviews) other than the standard paper and pencil tests to assess a student’s understanding of the material.



One method that has continued to increase student achievement is involving them in all steps of the assessment process. At the most basic level, students should understand how their grades will be determined. As assessment becomes more student-centered, the students can develop rubrics, maintain their own assessment records, self-assess, and communicate their achievement to others (student-led conferences).

PROCESS

The Ohio Learning Standards, which are the basis of the Diocese of Columbus' Course of Study, were developed through a feedback and revision process.

After the learning standards were adopted by the State Board of Education, a Diocesan committee was formed to review and adjust these standards so that the Course of Study reflects and integrates the Catholic faith and traditions.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and

both are assessable using mathematical tasks of sufficient richness.

DIFFERENTIATION

The content standards are grade and course-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should understand the standards and sequential order of mathematics as to allow for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of all students. For example, schools should allow students with a visual disability to use Braille, audio technology or other assistive devices for reading and a scribe, or speech-to-text technology for writing. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 8 with the Diocesan eight Standards for Mathematical Practices for Students which were adapted from the Ohio Mathematical Practices to include the Catholic virtues. Also included are a set of Teaching Practices from the National Council of Teachers of Mathematics (NCTM) on page 7.

Effective Mathematics Teaching Practices ²	
Establish mathematics goals to focus learning.	Effective teaching of mathematics establishes clear learning goals, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving.	Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allows for multiple entry points and varied solution strategies.
Use and connect mathematical representations.	Effective teaching of mathematics engages students in making connections among mathematical representations, deepening understanding of mathematics concepts and procedures. A Catholic math class should be a place where students learn to rejoice in God's truth.
Facilitate meaningful mathematical discourse.	Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing approaches and arguments. As they work together, students should exhibit the virtue of magnanimity or benevolence, keeping the greater good in the forefront.
Pose purposeful questions.	Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding.	Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, becomes skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics.	Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking.	Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning (e.g. "Show me your work!")

Mathematical Practices for Students with Connections to Catholic Virtues

Make sense of problems and persevere in solving them. Mathematically proficient students strive to understand the meaning of a problem and look for ways to solve it. They monitor, and evaluate their progress and change course if necessary. They conduct error analysis and learn from their mistakes. They demonstrate the virtues of perseverance as they work through the problem. They check their answers to problems using different methods and they continually ask themselves, “Does this make sense?”

Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They understand that symbols are used to represent quantities. They can represent a problem symbolically and manipulate the representing symbols to solve the problem. They can make connections between mathematical representations and the physical world. They understand that those representations reveal truths about creation and about God.

Construct viable arguments and critique the reasoning of others. Mathematically proficient students can listen to the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. They can justify their conclusions, communicate them to others, and respond to the arguments of others. They can use logic and reasoning to compare the effectiveness of two plausible arguments. If there is a flaw in an argument, they can use reasoning and logic to explain what it is. They demonstrate the virtues of temperance and humility in making their arguments and critiquing others.

Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can use mathematics to advance Catholic social teachings.

Mathematical Practices for Students (cont'd)

Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts when appropriate.

Attend to precision. Mathematically proficient students communicate precisely to others. They use clear definitions and mathematical language in discussion with others and in their own reasoning. They demonstrate self-control and humility in their communications.

Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. They also step back for an overview and shift perspective when needed. They understand that math is orderly and intelligible and a way to reveal truths about God and creation. They demonstrate the virtue of circumspection as they ponder the truths of creation.

Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for appropriate shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. They continue to demonstrate the virtues of prudence, fortitude and temperance while solving problems.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. A lack of understanding of the content effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice.

Noteworthy Changes From 2018 To 2024

There have been some changes to the Mathematics Course of Study in 2024. These changes are detailed out below:

Geometry

1. Congruence and proof of triangles were separated from construction and transformations as a separate critical area of focus.
2. Understanding and using line relationships was added.
3. Completing the square to find the center and radius of a circle was added as an extension to GPE.1.
4. Law of Sines and Cosines were added from Algebra II.

Algebra I

1. The Real Number standards (RN.1, RN.2 and RN.3) have been moved back into the Algebra I (from Algebra II) course of study with this additional note: The focus of these standards should be on the properties of square roots, simplifying square roots and the arithmetic operations of square roots (add, subtract, multiply).

Algebra II

1. Using the Law of Sines and Cosines to solve problems and to find unknown measurements in right triangles were moved to Geometry; however, finding unknown measurements in non-right triangles and extending right triangle trigonometry to include obtuse angles was left in Algebra II.
2. *Extend to include solving systems of linear equations using matrices, with the option to solve with technology* was added as an extension to A.REI.11.

Mathematical Content Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students. However, standards with a (+) symbol are considered over and above what is expected for that course.

The high school standards are listed in conceptual categories:

- Modeling
- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Proofs in high school mathematics should not be limited to geometry. Mathematically proficient high school students employ multiple proof methods, including algebraic derivations, proofs using coordinates, and proofs based on geometric transformations, including symmetries. These proofs are supported by the use of diagrams and dynamic software and are written in multiple formats including not just two-column proofs but also proofs in paragraph form, including mathematical symbols. In statistics, rather than using mathematical proofs, arguments are made based on empirical evidence within a properly designed statistical investigation.

How To Read The High School Content Standards

Conceptual Categories are areas of mathematics that cross through various course boundaries.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Standards define what students should understand and be able to do.

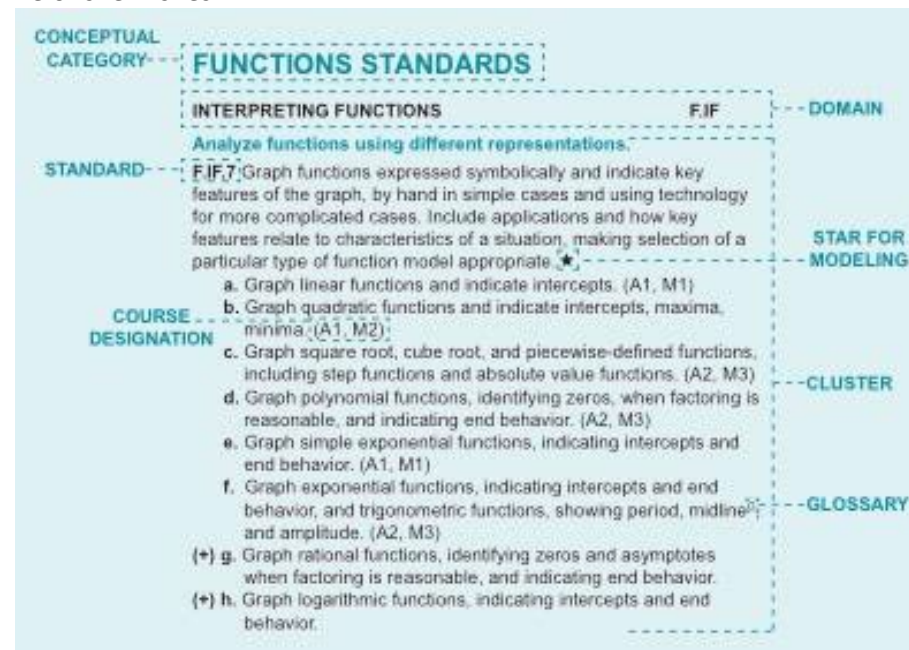
^G shows there is a definition in the glossary for this term.

(★) indicates that modeling should be incorporated into the standard.
(See the Conceptual Category of Modeling pages 12-13)

(+) indicates that it is a standard for students who are planning on taking advanced courses. Standards with a (+) sign will not appear on Ohio's State Tests.

Some standards have course designations such as (A1, M1) or (A2, M3) listed after an **a.**, **b.**, or **c.** These designations help teachers know where to focus their instruction within the standard. In the example below the beginning section of the standard is the stem. The stem shows what the teacher should be doing for all courses. (Notice in the example below that modeling (★) should also be incorporated.) Looking at the course designations, an Algebra 1 teacher should be focusing his or her instruction on **a.** which focuses on linear functions; **b.** which focuses on quadratic functions; and **e.** which focuses on simple exponential functions. An Algebra 1 teacher can ignore **c.**, **d.**, and **f.** as the focuses of these types of functions will come in later courses. However, a teacher may choose to touch on these types of functions to extend a topic if he or she wishes.

shows that the full extent of the stem is intended for an Algebra 2 or Math 3 course. However, **a.** shows that Algebra 1 and Math 2 students are responsible for a modified version of the stem that focuses on transformations of quadratic functions and excludes the $f(kx)$ transformation. However, again a teacher may choose to touch on different types of functions besides quadratics to extend a topic if he or she wishes if he or she wishes.



Notice that in the standard below, the stem has a course designation. This

High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or

investment growth.

- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either

improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO₂ over time.

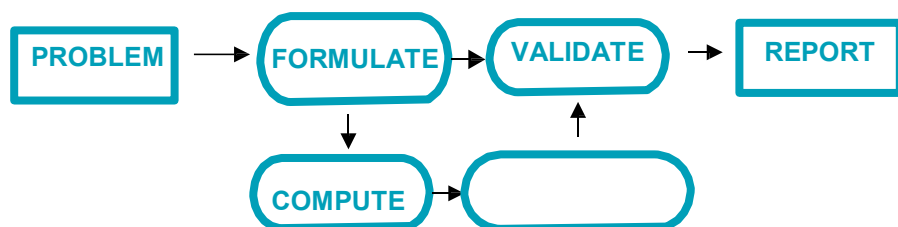
Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as

pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

MODELING STANDARDS

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★)



Geometry Critical Areas of Focus

In Geometry, instructional time should focus on seven critical areas:

CRITICAL AREA OF FOCUS 1: Applications of Probability

Building on probability concepts that began in grade 7, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions related to real-world situations.

CRITICAL AREA OF FOCUS 2: Construction and Transformation

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent or to have symmetries of itself, rotational or reflected. They apply reasoning to complete geometric constructions and explain why they work. Students will extend prior experience with geometric shapes toward the development of a hierarchy of two-dimensional figures based on formal properties.

CRITICAL AREA OF FOCUS 3: Similarity, Proof, and Trigonometry

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use it as a familiar foundation for the development of informal and formal proofs, problem solving and applications to similarity in right triangles. This will assist in the further development of right triangle trigonometry, with particular attention to special right triangles, right triangles with one side and one acute angle given and the Pythagorean Theorem. Students apply geometric concepts to solve

real-world, design and modeling problems.

CRITICAL AREA OF FOCUS 4: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

CRITICAL AREA OF FOCUS 5: Circles with and Without Coordinates

Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. Students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done with systems of equations in the first course to determine intersections between lines and circles. Students model and solve real-world problems applying these geometric concepts.

CRITICAL AREA OF FOCUS 6: Extending to Three Dimensions

Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Students develop the understanding of how changes in dimensions result in similar and non-similar shapes and how scaling changes lengths, areas and volumes. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. They solve real-world problems applying these geometric concepts.

CRITICAL AREA OF FOCUS 7: Congruence and Proof

Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal and informal proof. Students prove theorems—using a variety of formats—and apply them when solving problems about lines (such as parallel and perpendicular), triangles, quadrilaterals, and other polygons.

Geometry Course Overview

GEOMETRY

CONGRUENCE

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems both formally and informally using a variety of methods.
- Make geometric constructions.
- Classify and analyze geometric figures.
- Understand and use line relationships.

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

- Understand similarity in terms of similarity transformations.
- Prove and apply theorems involving similarity both formally and informally using a variety of methods.
- Define trigonometric ratios, and solve problems involving right triangles.

CIRCLES

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.

MODELING IN GEOMETRY

- Apply geometric concepts in modeling situations.

EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS

- Translate between the geometric description and the equation for a circle.
- Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

GEOMETRIC MEASUREMENT AND DIMENSION

- Explain volume formulas, and use them to solve problems.
- Visualize relationships between two- dimensional and three- dimensional objects.
- Understand the relationships between lengths, area, and volumes.

STATISTICS AND PROBABILITY

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

- Understand independence and conditional probability, and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means

the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric

understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

CONNECTIONS TO EQUATIONS

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof

Geometry Standards

CONGRUENCE

Experiment with transformations in the plane.

G.CO.1 Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.

G.CO.2 Represent transformations in the plane using, e.g.,

transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.

G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.

a. Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes.

b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

G.CO.6 Use geometric descriptions of rigid motions^G to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent^G.

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems both formally and informally using a variety of methods.

G.CO.9 Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

G.CO.10 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

G.CO.11 Prove and apply theorems about parallelograms. *Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Geometry Standards, continued

CONGRUENCE, CONTINUED

Make geometric constructions.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Classify and analyze geometric figures.

G.CO.14 Classify two-dimensional figures in a hierarchy based on properties.

Understand and use line relationships.

G.CO.15 Understand and use line relationships.

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

Understand similarity in terms of similarity transformations.

G.SRT.1 Verify experimentally the properties of dilations^G given by a center and a scale factor:

- A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G.SRT.2 Given two figures, use the definition of similarity in terms of

similarity transformations^G to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove and apply theorems both formally and informally involving similarity using a variety of methods.

G.SRT.4 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.

Define trigonometric ratios, and solve problems involving right triangles.

G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

G.SRT.8 Solve problems involving right triangles. ★

- Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)

G.SRT.9 Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.

G.SRT.10 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right triangles.

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.

High School - Geometry Standards, CONTINUED

CIRCLES

Understand and apply theorems about circles.

G.C.1 Prove that all circles are similar using transformational arguments.

G.C.2 Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

Understand and apply theorems about circles.

G.C.3 Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.

(+) **G.C.4** Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

G.C.5 Find arc lengths and areas of sectors of circles.

- Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. (G, M2)
- Derive the formula for the area of a sector, and use it to solve problems. (G, M2)

EXPRESSING GEOMETRIC PROPERTIES WITH EQUATION

Translate between the geometric description and the equation for a circle.

G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; (+) complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2)

G.GPE.5 Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.

G.GPE.6 Find the point on a directed line segment between two given points that partition the segment in a given ratio.

G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

GEOMETRIC MEASUREMENT AND DIMENSION

Explain volume formulas, and use them to solve problems.

G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Understand the relationships between lengths, areas, and volumes.

G.GMD.5 Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.

G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k , the effect on lengths, areas, and volumes is that they are multiplied by k , k^2 , and k^3 , respectively.

MODELING WITH GEOMETRY

Apply geometric concepts in modeling situations.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder. ★

G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot. ★

G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratio

HIGH SCHOOL—STATISTICS AND PROBABILITY

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media

and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

CONNECTIONS TO FUNCTIONS AND MODELING

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Standards

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

Understand independence and conditional probability, and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★

S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

S.CP.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having a dog and being a dog lover with the chance of being a cat lover and having a cat.* ★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.★

S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.★

(+) **S.CP.8** Apply the general Multiplication Rule in a uniform probability model^G, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.★ (G, M2)

(+) **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.★ (G, M2)

HIGH SCHOOL MATH APPENDIX- CURRICULUM INTEGRATION

1. History of a mathematician
 - a. Investigate past or current mathematician with a focus on saint examples
 - b. Include historical background
 - c. State their contributions.
 - d. Give their mathematical & historical importance.
 - e. How do we see their work used today.
 - f. Students share why they picked the person, and what was the most valuable thing they learned.
2. Vocabulary Scavenger Hunt
 - a. Terms are posted around the room.
 - i. Students must come up with images to illustrate.
 - ii. Students must also define the terms.
 - b. Images are provided to students.
 - i. Students find and name math found in the images.
 - ii. Students must also define the terms.
3. Financial Growth/Decay situations
 - a. Study the decreasing value of a car.
 - b. Study credit cards & mortgages for real value price paid.
 - c. Students find how much to invest at current rates in order to save for something in the near future (i.e. car, trip, etc.).
 - d. Study population growth and its impact on our world today.
4. Project involving the Golden Ratio
5. Project involving math in art
6. Project involving math in nature

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or

1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$.

Adjacent Angles.

Algorithm. *See also:* computation algorithm.

Associative property of addition. *See* Table 3 in this Glossary.

Associative property of multiplication. *See* Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football

team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.¹ *See also:* first quartile and third quartile.

Circumference.

Clusters. A number of similar things growing, collected or grouped together. Clusters can be used to break large collections of data into smaller groups.

Commutative property. *See* Table 3 in this Glossary.

Complex fraction. A fraction $\frac{A}{B}$ where A and/or B are fractions (B nonzero).

Complementary Angles.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* **computation strategy.**

Computation strategy. Purposeful manipulations that may be chosen for

specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

Dependent Variable. A variable that changes when the value of another variable changes. It is something that depends on other factors.

Diameter.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. **the process of resizing an object by a increasing or**

¹ Adapted from Wisconsin Department of Public Instruction, <http://dpi.wi.gov/standards/mathglos.html>, accessed March 2, 2010.

² Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. *See* Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

⁴ Adapted from Wisconsin Department of Public Instruction, *op. cit.*

decreasing the dimensions by certain scale factor can be “scaled” up or down

Discount.

Dividend. The number or quantity being divided (split into groups)

Divisor. The quantity that determines the number of equal parts or groups the dividend is split into

Dot plot. *See also:* line plot.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, $643 = 600 + 40 + 3$.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

Expression. A math phrase that contains numbers, operations, and/or variables.

Equation. Two or more expressions that are equal to one another.

Factor.

First quartile. For a data set with median M , the first quartile is the median of the data values less than M . Example:

For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fluency in algorithms -. The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

Fluency in math facts- Be able to reclassify math facts with automaticity.

Fluently. *See also:* fluency.

Fraction. A number expressible in the form $\frac{a}{b}$ where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Frequency.

Gaps. An interval where there are no data points present.

Identity property of 0. *See* Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the

combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Independent Variable. A variable that does not depend on any other variable for its value. It stands alone and isn't changed by the other variables you are trying to measure.

Inequality. A relationship between two expressions or values that are not equal to each other.

Integer. A number expressible in the form a or $-a$ for some whole number a .

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is $15 - 6 = 9$. *See also:* first quartile, third quartile.

Inverse. One of a pair of numbers that when operated on together give the identity.

Justify: To provide a convincing argument for the truth of a statement to a particular audience.

Linear Equation. An algebraic equation where each variable is raised to the power of 1. In one or two variables, it always represents a straight line.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.³

Markup.

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of

values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Mode.

Multiple.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 \div 8 = 9$.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $3/4$ and $4/3$ are multiplicative inverses of one another because $3/4 \times 4/3 = 4/3 \times 3/4 = 1$.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Outlier.

Perfect Square.

Pi.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $5/50 = 10\%$ per year.

Population.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is

used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

Proportional.

Prove: To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

Quotient. The answer to a division problem

Radius.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator in the form a/b or $-a/b$ for some fraction a/b . The rational numbers include the integers.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b . The rational numbers include the integers.

Reciprocal. One of a pair of numbers or

expressions whose product is one. See also **multiplicative inverse**.

Rectilinear figure. A polygon all angles of which are right angles.

Reflection.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. See also: terminating decimal.

Rotation.

Sample Population.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scaled Drawing.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on

a scatter plot.²

Similarity transformation. A rigid motion followed by a dilation.

Skewed Data.

Supplementary Angles.

Symmetrical Data.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to

measurement of other quantities as well.

Transversal.

Trapezoid. 1. A trapezoid is a quadrilateral with at least one pair of parallel sides (Inclusive definition) 2. A trapezoid is a quadrilateral with exactly one pair of parallel sides. (exclusive definition) *Districts may choose either definition to use for instruction. Ohio's State Tests' items will be written so that either definition will be acceptable.*

Uniform probability model. A probability model which assigns equal probability to all outcomes. See also: probability model.

Unit Rate. A ratio (a comparison using division) of different units whose denominator is one. Common Unit rate Miles per Gallon or Dollars per Hour

Variable. A variable is an object, event, idea, feeling, time period, or any other type of category you are trying to measure.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Verify: To check the truth or correctness

of a statement in specific cases.

Vertical Angles.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3,

² Adapted from Wisconsin Department of

Public Instruction