Mathematics
Course of Study K-8
2024
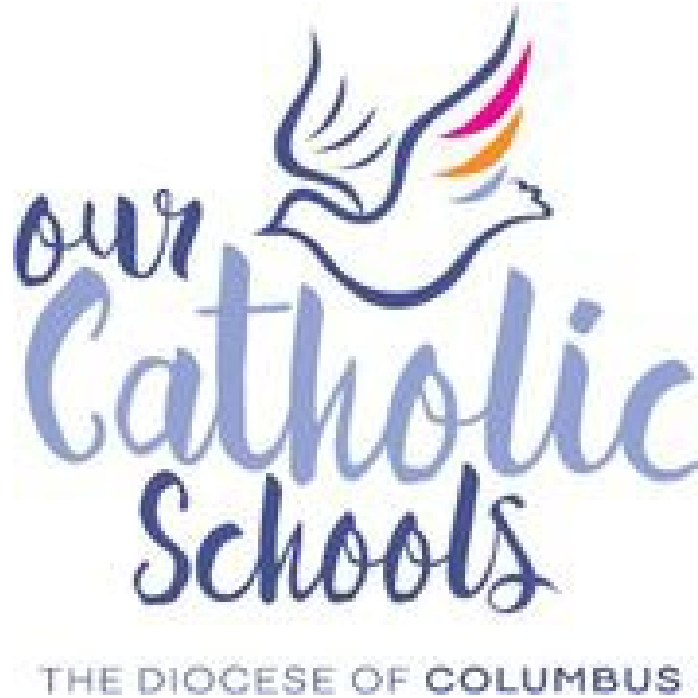

THE DIOCESE OF COLUMBUS

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INTRODUCTION

The following is the revised Mathematics Course of Study for the Catholic Diocese of Columbus. The committee has used the new Ohio Learning Standards for mathematics adopted by the State of Ohio in 2017 as the foundation of this Course of Study.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO OUR FAITH

The goal of mathematics education in a Catholic school is to produce mathematically literate, faithful individuals who can not only function in a global world of increasing complexity but who can also, more importantly, understand and speak the truth of God's teachings. To meet this challenging goal, the students in the Diocese of Columbus Catholic Schools will need to develop math content knowledge, apply their knowledge and skills, reason logically, think critically, solve problems creatively, resourcefully and morally, and communicate with others effectively, all within the context of our faith.

As Dr. Brett Salkeld states in his book, *Educating for Eternity*, "… an authentic Catholic education forms people who can change the world, not in spite of their desire for God and their hope for heaven, but because of it."[1] The need to understand and to be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. Those who accomplish this goal will significantly enhance their opportunities for shaping their future and the future of others. In keeping with this goal, students are presented with a sequential development of mathematical concepts. "Math is a unique and privileged way of perceiving the truth, goodness, and beauty of God's creation. It is therefore, a path to God himself."[1]

To best support the students, the classroom environment should foster enthusiastic learning and appreciation for the power, beauty, and usefulness of mathematics. Students will see mathematics as "both deeply mysterious and deeply intelligible,"[1] like God himself. They will see math reveal truths about creation and in doing so, reveals truth about the Creator. Math is an interdisciplinary problem-solving tool, a universal language, an art, and a powerful mechanism to further God's plan for humankind.

Effective teaching in a Catholic school engages students in application and cultivation of virtue into the math practices. Below are listed some examples of how a student can practice the virtues in mathematics.

Student will practice <u>perseverance</u>
- Continuing to work, willingly, even when the work is hard
- "Let's work through it step by step"
- "You can do hard things" (Philippians 4:13 - I can do all things through Christ which strengthens me)
- "We do not have a 'magic wand' for everything, but we do have trust in the Lord who accompanies us and never abandons us" (Pope Francis)

Students will practice <u>docility</u>
- Error analysis
- Graciously accepting feedback
- Proverbs 9:9 -"Instruct a wise man and he will be wiser still; teach a righteous man and he will add to his learning."

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[1] Salkeld, Brett. *Eucating for Eternity, A Teacher's Companion for Making Every Class Catholic*. Our Sunday Visitor, 2023.

Student will practice <u>humility</u>

- Being able to recognize that you need help and asking for help
- Not bragging that things are "too easy" and recognizing that others may need more time.
- Ephesians 4:2 – "Be completely humble and gentle; be patient, bearing with one another in love"

Student will practice <u>meekness</u>

- Staying in control of your actions and attitude
- Knowing how your attitude and actions impact others
- James 4:6- "But He gives a greater grace. Therefore, it says, "God is opposed to the proud, but gives grace to the humble.""

PRINCIPLES FOR MATHEMATICS FOR THE DIOCESE OF COLUMBUS

Equity. Excellence in mathematics education requires equity – high expectations based on the standards which should be accessible to all students, regardless of learning differences.

Curriculum. A curriculum is more than a collection of activities. It must be coherent, focused, well-articulated, and integrated with our Catholic values.

Teaching. Effective mathematics teaching requires a student-teacher relationship. Educators understanding what students know and need to learn and be able to do, and are supportive in moving students along the continuum of learning.

Learning. Students not only learn but understand mathematics with by actively building new knowledge from prior knowledge and experiences.

Technology. Technology is useful in teaching and learning mathematics. As any tool, technology can enhance students' learning while not substituting for the teacher and the student-teacher relationship.

Assessment. Multiple and appropriate assessments should align to the Course of Study and support the learning of important mathematics, be formative as well as summative, and furnish useful information to teachers, students and parents. Assessment results should guide teachers' instruction and interventions as well as grade promotion decisions. Useful assessments align to the standards in the Course of Study both in what a student needs to know and be able to do, and should match what the student is expected to learn. There are many tools (e.g. portfolios, rubrics, interviews) other than the standard paper and pencil tests to assess a student's understanding of the material.
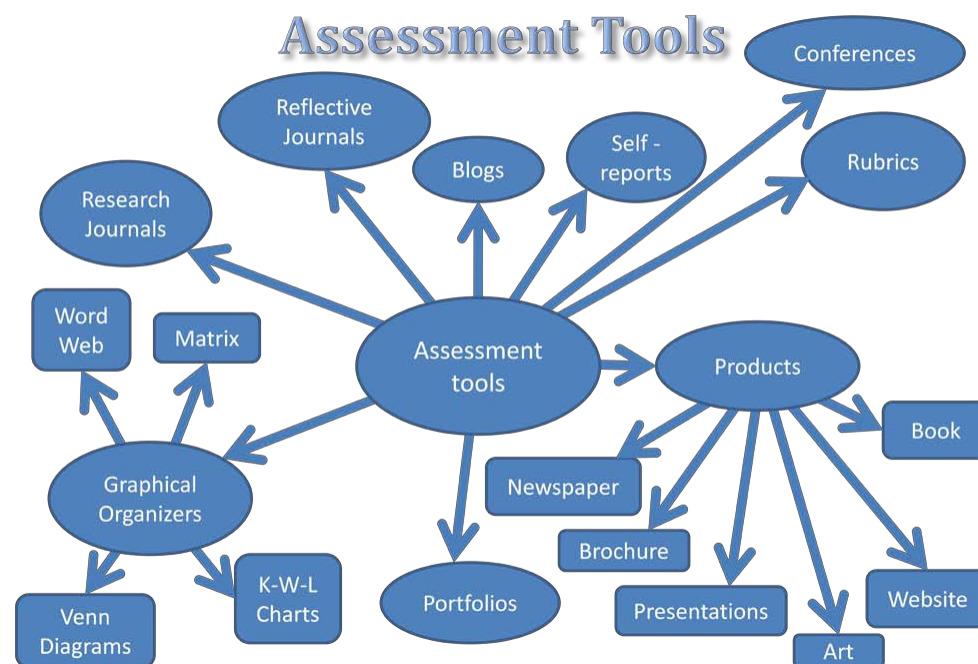
One method that has continued to increase student achievement is involving them in all steps of the assessment process. At the most basic level, students should understand how their grades will be determined. As assessment

becomes more student-centered, the students can develop rubrics, maintain their own assessment records, self-assess, and communicate their achievement to others (student-led conferences).

PROCESS

The Ohio Learning Standards, which are the basis of the Diocese of Columbus' Course of Study, were developed through a feedback and revision process.

After the learning standards were adopted by the State Board of Education, a Diocesan committee was formed to review and adjust these standards so that the Course of Study reflects and integrates the Catholic faith and traditions.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

DIFFERENTIATION

The content standards are grade and course-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should understand the standards and sequential order of mathematics as to allow for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of all students. For example, schools should allow students with a visual disability to use Braille, audio technology or other assistive devices for reading and a scribe, or speech-to-text technology for writing. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 9 with the Diocesan eight Standards for Mathematical Practices for Students which were adapted from the Ohio Mathematical Practices to include the Catholic virtues. Also included are a set of Teaching Practices from the National Council of Teachers of Mathematics (NCTM) on page 8.

Effective Mathematics Teaching Practices[2]

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear learning goals, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematicsengages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allows for multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations, deepening understanding of mathematics concepts and procedures. A Catholic math class should be a place where students learn to rejoice in God's truth.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing approaches and arguments. As they work together, students should exhibit the virtue of magnanimity or benevolence, keeping the greater good in the forefront.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, becomes skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistentlyprovides students, individually and collectively, with opportunities and supports to engage in productivestruggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of studentthinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning (e.g. "Show me your work!")

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[2] NCTM (National Council of Teachers of Mathematics)

Mathematical Practices for Students with Connections to Catholic Virtues

Make sense of problems and persevere in solving them. Mathematically proficient students strive to understand the meaning of a problem and look for ways to solve it. They monitor, and evaluate their progress and change course if necessary. They conduct error analysis and learn from their mistakes. They demonstrate the virtues of perseverance as they work through the problem. They check their answers to problems using different methods and they continually ask themselves, "Does this make sense?"

Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They understand that symbols are used to represent quantities. They can represent a problem symbolically and manipulate the representing symbols to solve the problem. They can make connections between mathematical representations and the physical world. They understand that those representations reveal truths about creation and about God.

Construct viable arguments and critique the reasoning of others. Mathematically proficient students can listen to the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. They can justify their conclusions, communicate them to others, and respond to the arguments of others. They can use logic and reasoning to compare the effectiveness of two plausible arguments. If there is a flaw in an argument, they can use reasoning and logic to explain what it is. They demonstrate the virtues of temperance and humility in making their arguments and critiquing others.

Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can use mathematics to advance Catholic social teachings.

Mathematical Practices for Students (cont'd)

Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts when appropriate.

Attend to precision. Mathematically proficient students communicate precisely to others. They use clear definitions and mathematical language in discussion with others and in their own reasoning. They demonstrate self-control and humility in their communications.

Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. They also step back for an overview and shift perspective when needed. They understand that math is orderly and intelligible and a way to reveal truths about God and creation. They demonstrate the virtue of circumspection as they ponder the truths of creation.

Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for appropriate shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. They continue to demonstrate the virtues of prudence, fortitude and temperance while solving problems.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. A lack of understanding of the content effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice.

NOTEWORTHY CHANGES FROM 2018 TO 2024

There have been some changes to the Mathematics Course of Study in 2024. These changes are detailed out below:

K-2 Grade Band
1. Patterns or repeated sequence standards were added back into the K-2 grade-band. Mathematics is built on patterns and young children have the ability to recognize and build upon simple patterns and should do so.
2. The counting on strategy will still be introduced in kindergarten but this strategy will not be expected to be mastered until the end of 1st grade. Cognitively, this is an abstract concept that is hard for most young children to grasp.

3. The recognition and identification of mathematical symbols (+, -, =) was added to kindergarten, both in isolation and in application.
4. The understanding of the meaning of the equal sign was moved to the end of 2nd grade. It will still be introduced in the 1st grade but not expected to be mastered till the end of 2nd grade. Cognitively, this is an abstract concept that is hard for most young children to grasp.
5. The introduction of non-standard measurement (e.g. paperclips and other objects) will be added to the kindergarten standards. It will continue to be expected that a student would master measuring with third objects in 1st grade.
6. We moved skip counting by 5 from 2nd grade to 1st grade.
7. We added the identification of nickels and quarter by name and value to 1st grade and added dollars by name and value to 2nd grade.

3-5 Grade Band
1. In grade 3, when covering properties of operations, students should be introduced to the property names; however, assessment should focus on application and not vocabulary.
2. The focus in 3rd grade is conceptual understanding of multiplication and division. Fluency standards were rewritten so that students are expected to fluently multiply and divide within 100 with automaticity at the end of 4th grade and within 144 (12x12) by the end of 5th.
3. 3rd grade will focus on telling time and elapsed time was moved to 4th grade.
4. We added students should know the relative sizes of US customary units to grade 4.
5. We added converting customary units and metric units to fifth grade.

6-8 Grade Band
1. We added more specificity and clarity to some of the standards.

HOW TO READ THE GRADE LEVEL STANDARDS

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

> *Number and Operations in Base Ten* → Domain
>
> *Use place value understanding and properties of operations to perform multi-digit arithmetic.* ← Cluster
>
> **3.NBT.1** Use place value understanding to round whole numbers to the nearest 10 or 100. → Standard
>
> **3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
>
> **3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
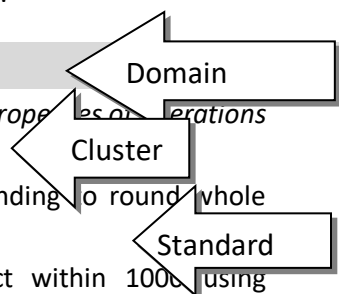
These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...."

But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity, therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards-based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

Kindergarten Critical Areas Of Focus

In Kindergarten, instructional time should focus on two critical areas:

Critical Area 1: Representing, relating, and operating on whole numbers, initially with sets of objects

Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 − 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

Critical Area 2: Describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics

Students describe their physical world using geometric ideas, e.g., shape, orientation, spatial relations, and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways, e.g., with different sizes and orientations, as well as three dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complicated shapes. They identify the measurable attributes of shapes.

KINDERGARTEN OVERVIEW

Counting and Cardinality
- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.
- <mark>Recognize simple patterns.</mark>

Operations and Algebraic Thinking
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten
- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data
- Identify, describe and compare measurable attributes.
- Classify objects and count the number of objects in each category.

Geometry
- Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
- Describe, compare, create, and compose shapes.

Kindergarten Standards

Counting and Cardinality   K.CC

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

K.CC.2 Count forward within 100 beginning from any given number other than 1.

K.CC.3 Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies.

- When counting objects, establish a one-to-one relationship by saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

- Understand that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted.

- Understand that each successive number name refers to a quantity that is one larger introducing "counting on" as a strategy.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

K.CC.6 Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group.

K.CC.7 Compare (without using inequality symbols) two numbers between 1 and 10 presented as written numerals.

K.CC.8 Recognize simple, repeated sequences and patterns (such as A,B,A,B or A,A,B,B …) and be able to predict the next term.

Operations and Algebraic Thinking   K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1a Represent addition and subtraction with objects, fingers, mental images, drawings, sounds such as claps, acting out situations, verbal explanations, expressions, or equations. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.1b Recognize and identify mathematical symbols (+, -, =) both in isolation and in application (e.g. equations).

K.OA.2 Solve addition and subtraction word problems (written or oral), and add and subtract within 10 by using objects or drawings to represent the problem.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings, and record each decomposition by a drawing or, when appropriate, an equation.

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or, when appropriate, equation.

K.OA.5 Fluently^G add and subtract within 5.

Number and Operations in Base Ten | **K.NBT**

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones by using objects or, when appropriate, drawings or equations; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data | **K.MD**

Describe and compare measurable attributes.

K.MD.1 Identify and describe measurable attributes of objects (length, weight and height) of a single object using vocabulary terms such as long/short, heavy/light, or tall/short.

K.MD.2a Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

K.MD.2b Begin to measure one item by using another item, e.g. paper clips and other objects (non-standard measurement).(Mastery in 1st grade)

Classify objects and count the number of objects in each category.

K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. The number of objects in each category should be less than or equal to ten. Counting and sorting coins should be limited to pennies.

Geometry | **K.G**

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind, and next to.*

K.G.2 Correctly name shapes regardless of their orientations or overall size.

K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three dimensional ("solid").

Analyze, compare, create, and compose shapes.

K.G.4 Describe and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their commonalities, differences, parts and other attributes.

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

K.G.6 Combine simple shapes to form larger shapes.

Grade 1 Critical Areas Of Focus

In Grade 1, instructional time should focus on four critical areas:

Critical Area 1: Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20.

Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models, e.g., cubes connected to form lengths, to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction, e.g., adding two is the same as counting on two. They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties, e.g., "making tens", to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

Critical Area 2: Developing understanding of whole number relationships and place value, including grouping in tens and ones.

Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes. Students use money as a tool to reinforce concepts of place value using pennies (ones) and dimes (tens).

Critical Area 3: Developing understanding of linear measurement and measuring lengths as iterating length units.

Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.

Critical Area 4: Reasoning about attributes of, and composing and decomposing geometric shapes.

Students compose and decompose plane or solid figures, e.g., put two triangles together to make a quadrilateral, and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

GRADE 1 OVERVIEW

Operations and Algebraic Thinking
- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten
- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data
- Measure lengths indirectly and by iterating length units.
- Work with time and money.
- Represent and interpret data.

Geometry
- Reason with shapes and their attributes.

Grade 1 Standards

Operations and Algebraic Thinking	1.OA

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. See Table 1, page 81.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards).

Understand and apply properties of operations and the relationship between addition and subtraction.

1.OA.3 Apply properties of operations as strategies to add and subtract. *For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition); to add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.* Students need not use formal terms for these properties.

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

1.OA.5 Relate counting to addition and subtraction e.g., by counting on^G 2 to add 2. (Mastery)

1.OA.6 Add and subtract within 20, demonstrating fluency^G for various strategies for addition and subtraction within 10. Strategies may include counting on; making ten e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14; decomposing a number leading to a ten e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9; using the relationship between addition and subtraction e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4; and creating equivalent but easier or known sums e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13.

Work with addition and subtraction equations.

1.OA.7 Begin to understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false with the traditional representation where the equal sign follows the operation. *For example, which of the following equations are true and which are false? 6 = 6, 5 +2 = 7, 10 – 5 = 5* (Mastery in 2nd grade)

1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + 6 = ?.*

Number and Operations in Base Ten	1.NBT

Extend the counting sequence.

1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral; skip-count by 5s starting from 0.

Understand place value.

1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a "ten;" the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones; and the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties

of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data	1.MD

Measure lengths indirectly and by iterating length units.

1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Work with time and money.

1.MD.3   Work with time and money.

    a.   Tell and write time in hours and half-hours using analog and digital clocks.

    b.   <mark>Identify pennies, dimes, nickels and quarters by name and value.</mark>

Represent and interpret data.

1.MD.4   Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry
1.G

Reason with shapes and their attributes.

1.G.1   Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

1.G.2   Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. <mark>Students do not need to learn formal names such as "right rectangular prism."</mark>

1.G.3   Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths*, and *quarters,* and use the phrases *half of, fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Grade 2 Critical Areas Of Focus

In Grade 2, instructional time should focus on four critical areas:

Critical Area 1: Extending understanding of base-ten notation.

Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones, e.g., 853 is 8 hundreds + 5 tens + 3 ones.

Critical Area 2: Building fluency with addition and subtraction.

Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds. They apply their understanding of addition and subtraction to data represented in the picture and bar graphs.

Critical Area 3: Using standard units of measure.

Students recognize the need for standard units of measure (centimeter and inch), and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length. They also apply number concepts solving real-world problems.

Critical Area 4: Describing and analyzing shapes.

Students describe and analyze shapes by examining their sides and angles.

Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades. They apply number concepts in real-world problems.

GRADE 2 OVERVIEW

Operations and Algebraic Thinking
- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data
- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry
- Reason with shapes and their attributes.

Grade 2 Standards

Operations and Algebraic Thinking	2.OA

Represent and solve problems involving addition and subtraction.

2.OA.1   Represent and solve problems involving addition and subtraction.

    a.   Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Table 1, page 81.

    b.   Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false regardless of where the equal sign falls. *For example, which of the following equations are true and which are false? 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 3 + 2. (*Mastery)

Add and subtract within 20.

2.OA.2   Fluently[G] add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. See standard 1.OA.6 for a list of mental strategies.

Work with equal groups of objects to gain foundations for multiplication.

2.OA.3   Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

2.OA.4   Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten	2.NBT

Understand place value.

2.NBT.1   Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

    a.   100 can be thought of as a bundle of ten tens - called a "hundred."

    b.   The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2   Count forward and backward within 1,000 by ones, tens, and hundreds starting at any number; skip-count by 5s starting at any multiple of 5.

2.NBT.3   Read and write numbers to 1,000 using base-ten numerals, number names, expanded form[G], and equivalent representations, e.g., 716 is 700 + 10 + 6, or 6 + 700 + 10, or 6 ones and 71 tens, etc.

2.NBT.4   Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

2.NBT.5   Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6   Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7   Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that in adding or subtracting three-digit numbers,

hundreds are added or subtracted from hundreds, tens are added or subtracted from tens, ones are added or subtracted from ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.8 Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.

Measurement and Data	2.MD

Measure and estimate lengths in standard units.

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same whole number units, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram[G] with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

2.MD.8 Solve problems with money.
  a. Identify dollars by name and value.
  b. Find the value of a collection of quarters, dimes, nickels, and pennies.
  c. Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ¢ symbols appropriately (not including decimal notation).

Represent and interpret data.

2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by creating a line plot[G], where the horizontal scale is marked off in whole-number units.

2.MD.10 Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put-together, take-apart, and compare problems in a graph. See Table 1, page 81 .

Geometry	2.G

Reason with shapes and their attributes.

2.G.1 Recognize and identify triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. Recognize and identify cubes, rectangular prisms, cones, and cylinders.

2.G.2   Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.G.3   Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves, thirds*, or *fourths* and *quarters,* and use the phrases *half of, third of,* or *fourth of* and *quarter of.* Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.

Grade 3 Critical Areas Of Focus

In Grade 3, instructional time should focus on five critical areas:

Critical Area 1: Developing understanding of multiplication and division and strategies for multiplication and division within 100.
Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

Critical Area 2: Developing understanding of fractions, especially unit fractions (fractions with numerator 1).
Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve

comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

Critical Area 3: Developing understanding of the structure of rectangular arrays and of area.
Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

Critical Area 4: Describing and analyzing two-dimensional shapes.
Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Critical Area 5: Solving multi-step problems.
Students apply previous understanding of addition and subtraction strategies and algorithms to solve multi-step problems. They reason abstractly and quantitatively by modeling problem situations with equations or graphs, assessing their processes and results, and justifying their answers through mental computation and estimation strategies. Students incorporate multiplication and division within 100 to solve multi-step problems with the four operations.

GRADE 3 OVERVIEW

Operations and Algebraic Thinking
- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten
- Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.

Number and Operations—Fractions
- Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Measurement and Data
- Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry
- Reason with shapes and their attributes.

Solve problems involving the four operations and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Numbers and Operations in Base Ten	3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1,000 using strategies and algorithms[G] based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g., 9×80, 5×60 using strategies based on place value and properties of operations.

Numbers and Operations - Fractions	3.NF

Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram[G].

  a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

  b. Represent a fraction $\frac{a}{b}$ (which may be greater than 1) on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

  a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.

  b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model[G].

  c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

  d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the

symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data	3.MD

Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1 Work with time and money.

    a. Tell and write time to the nearest <mark>minute using both analog and digital clocks.</mark> Measure time intervals in minutes (within 90 minutes).

    b. Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ¢ symbol appropriately (not including decimal notation).

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Table 2, page 81 .

Represent and interpret data.

3.MD.3 Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs. *For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.*

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot[G], where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

    a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

    b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

    c. Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths a and b + c is the sum of a × b and a × c (represent the distributive property with visual models including an area model).

    d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8   Solve real -world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry	3.G

Reason with shapes and their attributes.

3.G.1   Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).

3.G.2   Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

Grade 4 Critical Areas Of Focus

In Grade 4, instructional time should focus on three critical areas:

Critical Area 1: Developing an understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends as part of effectively and efficiently performing multi-digit arithmetic.

Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, and area models), place value, and properties of operations, particularly the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. Students efficiently and effectively add and subtract multi-digit whole numbers.

Critical Area 2: Developing an understanding of fraction equivalence, addition, and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.

Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal, e.g., $\frac{15}{9} =$ $\frac{5}{3}$, and they develop methods such as using models for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. Students solve measurement problems involving conversion of measurements and fractions.

Critical Area 3: Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, and particular angle measures.

Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems.

GRADE 4 OVERVIEW

Operations and Algebraic Thinking
- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- <mark>Fluently multiply and divide within 100.</mark>
- Generate and analyze patterns.

Number and Operations in Base Ten
- Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.
- Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.

Number and Operations—Fractions
- Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified).
- Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Measurement and Data
- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

Geometry
- Draw and identify lines and angles and classify shapes by properties of their lines and angles.

Grade 4 Standards

Operations and Algebraic Thinking	4.OA

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Table 2, page 81 . Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

4.OA.5 Fluently multiply and divide within 100 with automaticity.

Generate and analyze patterns.

4.OA.6 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Number and Operations in Base Ten	4.NBT

Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division.

4.NBT.2 Read and write multi-digit whole numbers using standard form, word form, and expanded form[G]. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.

4.NBT.4 Fluently[G] add and subtract multi-digit whole numbers using a standard algorithm[G].

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6 Find whole-number quotients and remainders with up to four digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between

multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations - Fractions   **4.NF**

Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.1   Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n}{n} \times \frac{a}{b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (e.g. $\frac{2}{2} \times \frac{3}{5} = \frac{6}{10}$)

4.NF.2   Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified).

4.NF.3   Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$.

     a.   Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

     b.   Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model[G]. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; 2 $\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

     c.   Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

     d.   Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4   Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

     a.   Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$ or $\frac{5}{4} = (\frac{1}{4}) + (\frac{1}{4}) + (\frac{1}{4}) + (\frac{1}{4}) + (\frac{1}{4})$.

     b.   Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$, recognizing this product as $\frac{6}{5}$. (In general, n $\times (\frac{a}{b}) = \frac{n \times a}{b}$.)

     c.   Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.5   Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express $\frac{3}{10}$

as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$. In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data	4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Solve problems involving measurement and conversion of measurements.

   a. Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter.

   b. Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds.

   c. Convert between hours, minutes and seconds.

4.MD.2 Solve real-world problems involving money, time, and measurement.

   a. Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock. (from 3rd)

   b. Using models, add and subtract money and express the answer in decimal notation.

   c. Using number line diagrams[G], clocks, or other models, add and subtract intervals of time in hours and minutes.

   d. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.

4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.

Represent and interpret data.

4.MD.4 Display and interpret data in graphs (picture graphs, bar graphs, and line plots[G]) to solve problems using numbers and operations for this grade.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

   a. Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1 /360 of a circle is called a "one-degree angle," and can be used to measure angles.

   b. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.

4.MD.6 Measure angles in whole number degrees using a protractor. Draw and measure angles to the nearest 5 degree (Ex- 5°,10°, 15°) Sketch angles of specified measure.

4.MD.7   Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry	*4.G*

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1   Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2   Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

Grade 5 Critical Areas Of Focus

In Grade 5, instructional time should focus on five critical areas:

Critical Area 1: Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)-Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They begin simplifying fractions using equivalent calculations. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. They apply their understanding of fractions to solve real-world problems. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Critical Area 2: Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations- Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

Critical Area 3: Developing understanding of volume Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

Critical Area 4: Modeling numerical relationships with the coordinate plane-Based on previous work with measurement and number lines, students develop understanding of the coordinate plane as a tool to model numerical relationships. These initial understandings provide the foundation for work with negative numbers, and ratios and proportional relationships in Grade Six and functional relationships in further grades.

Critical Area 5: Classifying two-dimensional figures by properties Students build on their understanding of angle measures and parallel and perpendicular lines to explore the properties of triangles and quadrilaterals. They develop a foundation for classifying triangles or quadrilaterals by comparing the commonalities and differences of triangles or between types of quadrilaterals.

GRADE 5 OVERVIEW

Operations and Algebraic Thinking
- Write and interpret numerical expressions.
- Analyze patterns and relationships.
- <mark>Fluently multiply and divide within 144.</mark>

Number and Operations in Base Ten
- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions
- Use equivalent fractions as a strategy to add and subtract fractions. (Simplification of fractions should be introduced[*]).
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Simplification of fractions should be introduced[*]).

Measurement and Data
- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry
- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Grade 5 Standards

Operations and Algebraic Thinking	5.OA

Write and interpret numerical expressions.

5.OA.1   Use parentheses in numerical expressions and evaluate expressions with this symbol. ==Limit formal use of algebraic order of operations to parenthesis, multiplication, division, addition and subtraction.==

5.OA.2   Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.*

Analyze patterns and relationships.

5.OA.3   Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

5.OA.4   ==Fluently multiply and divide within 144 with automaticity.==

Number and Operations in Base Ten	5.NBT

Understand the place value system.

5.NBT.1   Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.2   Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

5.NBT.3   Read, write, and compare decimals to thousandths.

a.   Read and write decimals to thousandths using base-ten numerals, number names, and expanded form[G], e.g., 347.392 = $3 × 100 + 4 × 10 + 7 × 1 + 3 × (\frac{1}{10}) + 9 × (\frac{1}{100}) + 2 × (\frac{1}{1000})$.

b.   Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.4   Use place value understanding to round decimals to any place, millions through hundredths.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5   Fluently[G] multiply multi-digit whole numbers using a standard algorithm[G].

5.NBT.6   Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.7   Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used.

a.   Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and

decimals through the hundredths place).

    b. Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place).

    c. Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). *For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.*

Number and Operations- Fractions	5.NF

Use equivalent fractions as a strategy to add and subtract fractions. (Simplification of fractions should be introduced).

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show 2 /3 + 5 /4 = 8 /12 + 15/12 = 23/12. In general, a /b + c /d = (a /b x d /d) + (c /d x b /b) = (ad + bc)/bd.*

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models[G] or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Simplification of fractions should be introduced).

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b}$ = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50 pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

    a. Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts, equivalently, as the result of a sequence of operations a × q ÷ b. *For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.)*

    b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.5 Interpret multiplication as scaling (resizing).

    a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

    b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given

number; and relating the principle of fraction equivalence $\frac{a}{b}$ = $\frac{nxa}{nxb}$ to the effect of multiplying $\frac{a}{b}$ by 1.

5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general, students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade.

  a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $\frac{1}{3} \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$.

  b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div \frac{1}{5}$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div \frac{1}{5} = 20$ because $20 \times \frac{1}{5} = 4$.*

  c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$ pound of chocolate equally? How many $\frac{1}{3}$ cup servings are in 2 cups of raisins?*

Measurement and Data   5.MD

Convert like measurement units within a given measurement system.

5.MD.1 Convert like measurement units within a given measurement system.

  a. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.

  b. Convert between metric units kilometer, meter, centimeter, millimeter; kilograms, gram; liter, milliliter.

5.MD.2 Display and interpret data in graphs (picture graphs, bar graphs, and line plots^{G}) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, or decimals.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

  a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

  b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

  a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products

as volumes, e.g., to represent the Associative Property of Multiplication.

b. Apply the formulas V = ℓ × w × h and V = B × h for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Geometry	5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., x-axis and x-coordinate, y-axis and y-coordinate.

5.G.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

5.G.3 Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral,

and scalene triangles).

5.G.4 Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids[G], and rhombuses.

Grade 6 Critical Areas Of Focus

In Grade 6, instructional time should focus on five critical areas:

Critical Area 1: Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems Students use reasoning about multiplication and division to solve ratio and rate problems about quantities.

By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

Critical Area 2: Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers

Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

Critical Area 3: Writing, interpreting, and using expressions and equations Students understand the use of variables in mathematical expressions.

They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems.

Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

Critical Area 4: Developing understanding of statistical problem-solving Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically.

The GAISE model is used as a statistical problem-solving framework. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (range and interquartile range) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, gaps, peaks, and outliers in a distribution, considering the context in which the data were collected.

Critical Area 5: Solving problems involving area, surface area, and volume
Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students

discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plan.

GRADE 6 OVERVIEW

Ratio and Proportional Relationships
- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System
- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations
- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry
- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability
- Develop understanding of statistical problem solving.
- Summarize and describe distributions.

Grade 6 Standards

Ratios and Proportional Relationships	6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a unit rate of $5 per hamburger."*

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams[G], double number line diagrams[G], or equations. *An example would be uses of the golden ratio specifically in art and God's creation.*

   a. Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

   c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent.

   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System	6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models[G] and equations to represent the problem. Highlight the relationship between the quotient and the divisor (e.g. when the divisor gets smaller, the quotient gets bigger). Emphasize that multiplying by the reciprocal is the same as dividing by the original. *For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 pound of chocolate equally? How many 3/4 cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.2 Fluently[G] divide multi-digit numbers using a standard algorithm[G].

6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.

6.NS.5 Understand that positive and negative numbers are used together

to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., − (−3) = 3, and that 0 is its own opposite.

   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.7 Understand ordering and absolute value of rational numbers.

   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right.

   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write −3°C > −7°C to express the fact that −3°C is warmer than −7 °C.

   c. Understand the absolute value of a rational number as its

distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of −30 dollars, write |−30| = 30 to describe the size of the debt in dollars.

   d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars.

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations	*6.EE*

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole number exponents.

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 − y.*

   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those

involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas V = s³ and A = 6s² to find the volume and surface area of a cube with sides of length s = 1 /2.*

6.EE.3   Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

6.EE.4   Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

6.EE.5   Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.6   Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.7   Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.

6.EE.8   Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem.

Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9   Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Geometry	*6.G*

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1   Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.

6.G.2   Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.3   Draw polygons in the coordinate plane given coordinates for the

vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.4   Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real world and mathematical problems.

Statistics and Probability	6.SP

Develop understanding of statistical problem solving.

6.SP.1   Develop statistical reasoning by using the GAISE model:

  a.   Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.* (GAISE Model, step 1)

  b.   Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)

  c.   Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)

  d.   Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)

6.SP.2   Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.3   Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.4   Display numerical data in plots on a number line, including dot plots[G] (line plots), histograms, and box plots[G]. (GAISE Model, step 3)

6.SP.5   Summarize numerical data sets in relation to their context.

  a.   Report the number of observations.

  b.   Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.

  c.   Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range[G]) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution.

  d.   Choose the measures of center and variability, based on the shape of the data distribution (i.e. symmetrical vs. skewed) and the context in which the data were gathered.

Grade 7 Critical Areas Of Focus

In Grade 7, instructional time should focus on five critical areas:

Critical Area 1: Developing understanding of and applying proportional relationships

Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

Critical Area 2: Developing understanding of operations with rational numbers and working with expressions and linear equations

Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts, e.g., amounts owed or temperatures below zero, students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

Critical Area 3: Solving problems involving scale drawings and informal geometric constructions, angles, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume

Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Critical Area 4: Drawing inferences about populations based on samples

Students build on their previous work with statistical problem solving through the use of the GAISE model framework. They summarize and describe distributions representing one population and informally compare two populations. Students interpret numerical data sets using mean absolute deviation. They begin informal work with sampling to generate data sets: learn about the importance of representative samples for drawing inferences and the impact of bias.

Critical Area 5: Investigating chance

Students build upon previous understandings as they develop concepts of probability. They investigate relevant chance events and develop models to determine and compare probabilities. They analyze the frequencies of the experimental results against their predictions, justifying any discrepancies. Students extend their investigations with compound events representing the

possible outcomes in tree diagrams, tables, lists, and ultimately through designing and using simulations.

GRADE 7 OVERVIEW

Ratio and Proportional Relationships

- Analyze proportional relationships and use them to solve real world and mathematical problems.

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.

Statistics and Probability

- Use sampling to draw conclusions about a population.
- Broaden understanding of statistical problem solving.
- Summarize and describe distributions representing one population and draw informal comparisons between two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Grade 7 Standards

Ratios and Proportional Relationships	7.RP

Analyze proportional relationships and use them to solve real world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction^G (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.*

7.RP.2 Recognize and represent proportional relationships between quantities.

   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

   c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.

   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System	7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

   b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

   c. Understand subtraction of rational numbers as adding the additive inverse, p − q = p + (−q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real world contexts.

   d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then −(p /q) = (−p) /q = p /(−q). Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Expressions and Equations	7.EE

Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 In real-life applications, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by p − 0.15p) is equivalent to (1 − 0.15)p, which is equivalent to 0.85p or finding 85% of the original price.*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example, if a woman making $25 an hour gets a 10% raise, she will make an additional 1 /10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 ¾ inches long in the center of a door that is 27 1 /2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry	7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.

a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.

b. Represent proportional relationships within and between similar figures.

7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions.

a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

b. Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.

7.G.4 Work with circles.

a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle.

b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure <mark>of two intersecting lines.</mark>

7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability   **7.RP**

Use sampling to draw conclusions about a population.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population.

a. Differentiate between a sample and a population.

b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.

Broaden understanding of statistical problem solving.

7.SP.2 Broaden statistical reasoning by using the GAISE model:

a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1)*

b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)

c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)

d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)

Summarize and describe distributions representing one population and draw informal comparisons between two populations.

7.SP.3 Describe and analyze distributions.

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a. Summarize quantitative data sets in relation to their context by using mean absolute deviation[G] (MAD), interpreting mean as a balance point.

b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot[G] (line plot), the separation between the two distributions of heights is noticeable.*

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around 1 /2 indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.7 Develop a probability model[G] and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model[G] by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space[G] for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Grade 8 Critical Areas Of Focus

In Grade 8, instructional time should focus on four critical areas:

Critical Area 1: Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations
Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m·A. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables graphically or by simple inspection; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

Critical Area 2: Grasping the concept of a function and using functions to describe quantitative relationships
Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

Critical Area 3: Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem
Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Critical Area 4: Working with irrational numbers, integer exponents, and scientific notation
Students explore irrational numbers and their approximations. They extend work with expressions and equations with integer exponents, square and cube roots. Understandings of very large and very small numbers, the place

value system, and exponents are combined in representations and computations with scientific notation.

GRADE 8 OVERVIEW

The Number System
- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations
- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions
- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Statistics and Probability
- Investigate patterns of association in bivariate data

Grade 8 Standards

The Number System	8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions and Equations	8.EE

Work with radicals and integer exponents.

8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of 1-12 (perfect squares only) and cube roots of 1-5 (perfect cubes only). Know that $\sqrt{2}$ is irrational.

8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8; and the population of the world as 7×10^9; and determine that the world population is more than 20 times larger.*

8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations with one variable.
   a. Give examples of linear equations with one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.8 Analyze and solve pairs of simultaneous linear equations graphically.

    a. Understand that the solution to a pair of linear equations with two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously.

    b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations with two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.

    c. Solve real-world and mathematical problems leading to pairs of linear equations with two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)

Functions	8.F

Define, evaluate, and compare functions.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s² giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates).

    a. Lines are taken to lines, and line segments are taken to line segments of the same length.

    b. Angles are taken to angles of the same measure.

    c. Parallel lines are taken to parallel lines.

8.G.2 Understand that a two-dimensional figure is congruent[G] to another if the second can be obtained from the first by a sequence of

rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.)

8.G.3 Describe the effect of dilations[G], translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9 Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.

Statistics and Probability	8.SP

Investigate patterns of association in bivariate data.

8.SP.1 Construct and interpret scatter plots for bivariate[G] measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4)

8.SP.2 Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4)

8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. (GAISE Model, steps 3 and 4)

8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores.*

HIGH SCHOOL MATHEMATICS IN MIDDLE SCHOOL

Our Catholic schools strive to meet the academic needs of all students within our schools. It is educationally sound to accelerate a student when the student demonstrates the need and ability to move beyond the grade-level curriculum. There are some students who are able to move through the mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade or earlier so they can take college-level mathematics in high school. The number of students taking high school mathematics in eighth grade has increased steadily for years. Part of this trend is the result of a concerted effort to get more students to take Calculus and other college-level mathematics courses in high school. Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.

In order to meet the accelerated student's needs, diocesan schools have the option to offer Algebra 1 in 8th grade as well as the option to offer curriculum-compacted classes starting in 6th or 7th grade. There is a Diocesan Course of Study for both curriculum-compacted options. Policy 6216 outlines the criteria for selection of students and for teaching these courses.

Candidates will be assessed on the 8th grade content. Skipping material to get students to a particular point in the curriculum will likely create gaps in the students' mathematical background, which may create additional problems later. Students may miss important content from eighth-grade mathematics and that could result in being unprepared for higher level mathematics.

Assessing the students on critical 8th grade content will help alleviate that possibility.

Overview of the Accelerated Pathway

Starting in Sixth Grade (CC678)

Standards covered in 6th grade, 678-6	Standards Covered in 7th grade, 678-7
Ratios and Proportional Reasoning	
6.RP.1	8.EE.5
6.RP.2	
6.RP.3	
7.RP.1	
7.RP.2	
7.RP.3	
The Number System	
6.NS.1	8.NS.1
6.NS.2	8.NS.2
6.NS.3	
6.NS.4	
7.NS.1	
7.NS.2	
7.NS.3	
6.NS.5	
6.NS.6	
6.NS.7	
6.NS.8	
Expressions and Equations	
6.EE.1	8.EE.1
6.EE.2	8.EE.2
6.EE.3	8.EE.3
6.EE.4	8.EE.4
6.EE.5	8.EE.6
6.EE.6	8.EE.7
6.EE.7	8.EE.8
6.EE.8	
6.EE.9	

Standards covered in 6th grade, 678-6	Standards Covered in 7th grade, 678-7
Expressions and Equations (con't)	**Functions**
7.EE.1	8.F.1
7.EE.2	8.F.2
7.EE.3	8.F.3
7.EE.4	8.F.4
	8.F.5
Geometry	
6.G.1	7.G.5
6.G.3	7.G.4
6.G.2	8.G.1
6.G.4	8.G.2
	8.G.3
	8.G.4
	8.G.5
	7.G.1
	7.G.2
	7.G.3
	8.G.6
	8.G.7
	8.G.8
	8.G.9

Standards covered in 6th grade, 678-6	Standards Covered in 7th grade, 678-7
Statistics and Probability	
6.SP.1	8.SP.1
6.SP.2	8.SP.2
6.SP.3	8.SP.3
6.SP.4	8.SP.4
6.SP.5	7.SP.1
	7.SP.2
	7.SP.3
	7.SP.4
	7.SP.5
	7.SP.6
	7.SP.7
	7.SP.8

Starting in Seventh Grade (CC78-7)

Standards covered in 7th grade
Ratios and Proportional Reasoning
7.RP.1
7.RP.2
7.RP.3
8.EE.5
The Number System
7.NS.1
7.NS.2
7.NS.3
8.NS.1
8.NS.2
Expressions and Equations
7.EE.1
7.EE.2
7.EE.3
7.EE.4
8.EE.1
8.EE.2
8.EE.3
8.EE.4
8.EE.6
8.EE.7
8.EE.8
Functions
8.F.1
8.F.2
8.F.3
8.F.4
8.F.5

Geometry
7.G.1
7.G.2
7.G.3
7.G.4
7.G.5
8.G.1
8.G.2
8.G.3
8.G.4
8.G.5
8.G.6
8.G.7
8.G.8
8.G.9
Statistics and Probability
8.SP.1
8.SP.2
8.SP.3
8.SP.4
7.SP.1
7.SP.2
7.SP.3
7.SP.4
7.SP.5
7.SP.6
7.SP.7
7.SP.8

Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[4] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or

subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 − 5 = 9 is a subtraction within 20, and 55 − 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: ¾ and − $3/4$ are additive inverses of one another because ¾ + (−$3/4$) = (− $3/4$) + $3/4$ = 0.

Adjacent Angles.

Algorithm. *See also:* computation algorithm.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1] *See also:* first quartile and third quartile.

Circumference.

Clusters. A number of similar things growing, collected or grouped together. Clusters can be used to break large collections of data into smaller groups.

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Complementary Angles.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the

correct result in every case when the steps are carried out correctly. *See also:* **computation strategy**.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from

the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dependent Variable. A variable that changes when the value of another variable changes. It is something that depends on other factors.

Diameter.

Dilation. A transformation that moves each point

along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. the process of resizing an object by a increasing or decreasing the dimensions by certain scale factor can be "scaled" up or down

Discount.

Dividend. The number or quantity being divided (split into groups)

Divisor. The quantity that determines the number of equal parts or groups the dividend is split into

Dot plot. *See also:* line plot.

Expanded form. A multi- digit number is expressed in expanded form when it is written as a sum of single- digit multiples of powers of ten. For example,
643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

Expression. A math phrase that contains numbers, operations, and/or variables.

Equation. Two or more expressions that are equal to one another.

Factor.

First quartile. For a data set with median M, the first quartile is the median of
the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14,
15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fluency in algorithms -. The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

Fluency in math facts- Be able to reclass math facts with automaticity.

Fluently. *See also:*
fluency.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Frequency.

Gaps. An interval where there are no data points present.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Independent Variable. A variable that does not depend on any other variable for its value. It stands alone and isn't changed by the other variables you are trying to measure.

Inequality. A relationship between two expressions or values that are not equal to each other.

Integer. A number expressible in the form a or $-a$ for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 − 6 = 9. *See also:* first quartile, third quartile.

Inverse. One of a pair of numbers that when operated on together give the identity.

Justify: To provide a convincing argument for the truth of a statement to a particular audience.

Linear Equation. An algebraic equation where each variable is raised to the power of 1. In one or two variables, it always represents a straight line.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Markup.

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Mode.

Multiple.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Outlier.

Perfect Square.

Pi.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Population.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

Proportional.

Prove: To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

Quotient. The answer to a division problem

Radius.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator in the form a/b or − a/b for some fraction a/b. The rational numbers include the integers.

Rational number. A number expressible in the form a/b or − a/b for some fraction a/b. The rational numbers include the integers.

Reciprocal. One of a pair of numbers or expressions whose product is one. See also **multiplicative inverse**.

Rectilinear figure. A polygon all angles of which are right angles.

Reflection.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. See also: terminating decimal.

Rotation.

Sample Population.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scaled Drawing.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[3]

Similarity transformation. A rigid motion followed by a dilation.

Skewed Data.

Supplementary Angles.

Symmetrical Data.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Transversal.

Trapezoid. 1. A trapezoid is a quadrilateral with at least one pair of parallel sides (Inclusive definition) 2. A trapezoid is a quadrilateral wit exactly one pair of parallel sides. (exclusive definition*.)*

Uniform probability model. A probability model which assigns equal probability to all outcomes. See also: probability model.

Unit Rate. A ratio (a comparison using division) of different units whose denominator is one. Common Unit rate Miles per Gallon or Dollars per Hour

Variable. A variable is an object, event, idea, feeling, time period, or any other type of category you are trying to measure.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Verify: To check the truth or correctness of a statement in specific cases.

Vertical Angles.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….